Calculus…It's Fundamental

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Introduction and Rationale:

This AP Calculus lesson introduces students to the fundamental theorem of calculus. It is part of a two day lesson on the topic. As an introduction, students will explore the position of an object at given times when supplied with a position equation. Links will then be made to the area under the corresponding velocity curve. The generalized result is a major component of the fundamental theorem.

Objectives:

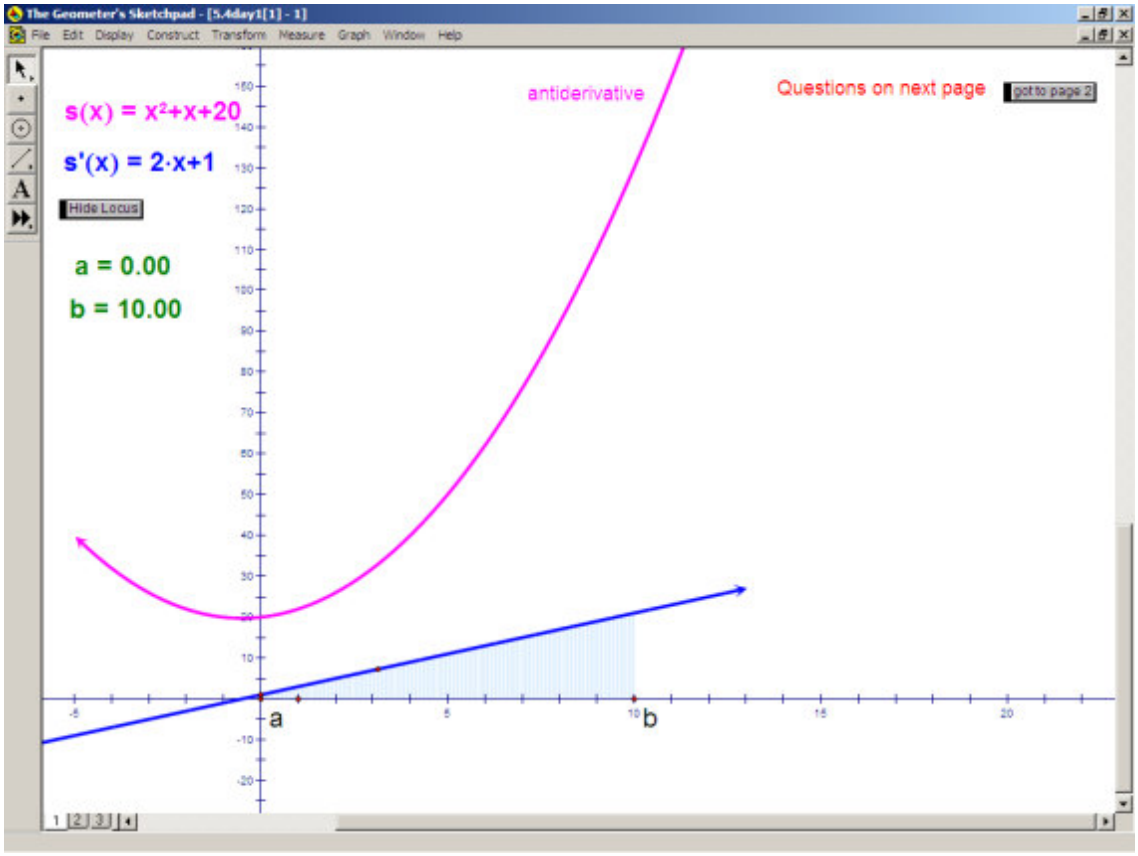
The students will

- Explore the relationship between integration and differentiation.
- Explore the connections between the area under a curve and the *y* coordinate of a related curve (the anti-derivative)
- Connect the concepts of velocity to position through the use of the antiderivative.
- Interpret and analyze constructed graphs
- Justify the formula: $\int_a^b s'(t)\,dt = s(b) - s(a)$

Learning Activities/Outline:

Students in this lesson first explore the position function and its derivative the velocity function using *Geometer's Sketchpad.* The pink graph represents the position versus time graph of an object. The blue graph is the rate of change of the pink graph. (In other words the blue graph is the representation of the velocity or derivative of the position function). The students begin by

answering some simple questions about the position function.



The formula $s(t) = t^2 + t + 20$ provides the position of a car in feet as a function of time t, for $t = 0$ to $t = 10$.

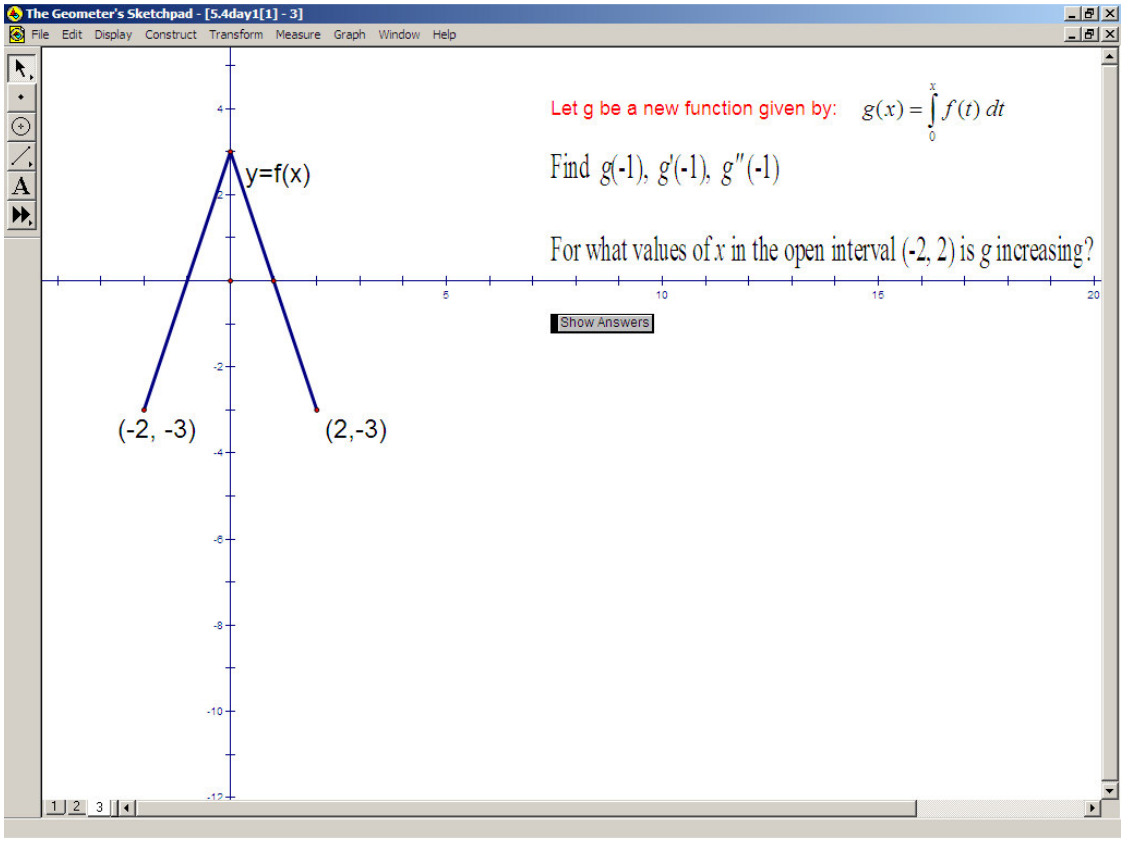
- What is the position of the car at t=0?
- What is the position of the car at t=10?
- What is the change in position of the car from $t = 0$ to $t = 10$
- Determine the value for $\int_0^{10} s'(t)\,dt$ (this can be done strictly geometrically too)

- What is the significance of this result? Explain why this happened?
- What is the cars change in position from $t = 2$ to $t = 12$ seconds?
- Calculate $\int_2^{12} s'(t)\,dt$ (this can be done strictly geometrically too)

In general: $\int_a^b s'(t)\,dt =$ _____

Using the patterns that are obvious through working in *Geometer's Sketchpad*, students are lead to the fascinating conclusion that the area under the blue curve from 0 to 10 is the same answer as the change in the height (y-value) of the pink curve when you move from 0 to 10. In fact, both answers represent the change in displacement of the moving object. Working in groups students will then connect their reasoning to the concept of an antiderivative of a function.

For the second part, students will be provided with the following graph and questions. The focus of this part of the discussion is upon the inverse relationship between the operations of integration and differentiation. Connections are made back to the concept of antiderivative. For example, $g'(x)$ must be $f(x)$. So $g(x)$ is the antiderivative of $f(x)$. Using this information would allow them to solve these questions. Several of these answers are connected to areas or slopes of segments. Both of these are easily measured with the computer software.



Laptop Implications:

The activities for this lesson were designed specifically for the laptops. Without the use of *Geometer's Sketchpad*, it would be extremely difficult to complete this project. The computers easily allow the students to visualize the connections between very difficult concepts.

Role of the Teacher: There is a great deal of annotation done by the instruction to facilitate the connections between the concepts. For example, the solution to the $\int_0^{10} s'(t)\,dt$ is as simple as calculating the area of a trapezoid. Through discussion and questioning, the students come to recognize this area as representing the

distance traveled. (If you draw a rectangle whose height is velocity-say ft/s- and whose width is time in seconds, their product (area) will be feet—distance) Because of the complexity of the material and the amount of annotation, the lessons are currently being recorded and available for student download at http://ww.uhigh.ilstu.edu/math/thompson/ipod/apcalc.xml or available through searching the podcast database through iTunes.

NCTM Standards:

Algebra Standard:

- **Analyze change in various contexts**
- **Use mathematical models to represent and understand quantitative relationships**