Investigating Graphs of Polynomial Functions

by Laura Sellers

Introduction:

This lesson is designed for a 2nd-year algebra class. It is a basic introduction to the graphs of polynomial functions.

Objectives:

- Students will use properties of end behavior to analyze, describe, and graph polynomial functions.
- Students will identify and use maxima and minima of polynomial functions to solve problems.
- Students will understand new terms including end behavior, turning point, local maximum, and local minimum.

Learning Activities:

1) Given the list of polynomial functions (see attached sheet #1), please do each of the following for each function:
   a) sketch graph
   b) state the function's degree
   c) state the number of turns in the graph

Students may use their graphing calculator or Geometer's Sketchpad.
2) Answer summary questions (see attached sheet #2).
3) Discuss box problem (see attached sheet #3).

Technology Implications:

The students will have access to their own graphing calculator and Geometer's Sketchpad to graph polynomial functions. They will use one-note on their laptop to write down patterns and discoveries. Student work will be displayed using Classroom Spy while they are presenting their ideas. Lastly, all activity sheets and the homework assignment will be accessed through Blackboard.

Illinois Learning Standards:

8.B.5 Use functions including exponential, polynomial, rational, parametric, logarithmic, and trigonometric to describe numerical relationships.
8.C.5 Use polynomial, exponential, logarithmic and trigonometric functions to model situations.
9.C.5b Apply physical models, graphs, coordinate systems, networks and vectors to develop solutions in applied contexts (e.g., bus routing, areas of irregular shapes, describing forces and other physical quantities).

Source:

Burger, E., Chard, D., J, H. E., Kennedy, P. A., Leinwand, S. J., Renfro, F. L., et al. (2007). *Algebra 2*. Austin: Holt, Rinehardt and Winston.

Polynomial Functions

For each function, sketch graph, state the degree, and give the number of turns in each graph.

a) $f(x) = 2x + 3$

b) $f(x) = -3x + 1$

c) $f(x) = x^2 + 2x - 5$

d) $f(x) = -x^2 - 5$

e) $f(x) = 2x^3 - 7x + 1$

f) $f(x) = -x^3 + 5x + 2$

g) $f(x) = x^4 - 2x^2 + 3$

h) $f(x) = -x^4 + 3x^3 + 2x^2 - 7x - 1$

i) $f(x) = x^5 - 5x^3 + 4x$

j) $f(x) = -x^5 + 5x^3 - 4x$

Summary Questions

1. What is the relationship between the degree and the number of turns?

2. Sketch the graph for $f(x) = x^4 - 3$. State the degree and the number of turns.

3. What is similar about the <u>even</u> degree functions? (c, d, g, h)

4. What is similar about the <u>odd</u> degree functions? (a, b, e, f, i, j)

5. What is the relationship between the number of zeros and the degree?

Box Problem

An artist plans to construct an open box from a 15 in. by 20 in. sheet of metal by cutting squares from the corners and folding up the sides. Find the maximum volume of the box and the corresponding dimensions.