1. Find the value of x for which $9-x=8-(2x-5)$.

2. The sum of twice a number and three times the same number is 135.6. Find the number. Express your answer as a **decimal**.

3. There are three consecutive **odd** integers. If five times the largest is 53 more than twice the smallest, find the largest of the three consecutive odd integers.

4. If $(5x-2)^2-(x-3)^2=3$ is written in the form $ax^2-7x+b=0$, find the value of $(a+b)$. Assume that a and b are integers.

5. Find the largest number which can be expressed as the product of positive integers (not necessarily distinct, but each of which is **greater than three**) if the sum of the positive integers is 31.

6. If $8^x=16$, find the value of x. Express your answer as an improper fraction reduced to lowest terms.

7. Dolly has exactly four coins: a penny, a dime, a quarter, and a half dollar. If one of Dolly's coins is selected at random, find the probability that the number of cents in that coin is an integral multiple of ten. Express your answer as a common fraction reduced to lowest terms.

8. Find the value of x for which $\frac{2x+3}{x-3}-1=\frac{12}{x+3}+1$.

9. How many times is the numeral 4 required in numbering the pages of a book from 1 to 1399?

10. If the product of the roots of $x^2-3x+17=0$ is added to 4 times the sum of the roots of the same equation, the resulting value is k. Find the value of k.

11. 87 is divided into three parts in the ratio of $1:\frac{2}{3}:\frac{3}{4}$. Find the value of the smallest part.

12. On a perfectly straight road on a perfectly flat surface, Steve starts riding his bike due south from point A at the constant rate of 15 mph. On the same road exactly three hours later, Monica starts riding her bike due south from point A at the constant rate of 18 mph. In how many hours after Monica starts will she catch Steve?

13. Find the value of k such that the equation $5x^2 - 12x + 3k = 0$ has exactly one solution for x.

14. The quadratic equation whose roots are the reciprocals of the roots of the equation $-37x^2 + 114x + 1 = 0$ can be written in the form $x^2 + kx + w = 0$. Find the value of $(k + w)$.

15. If $\frac{1}{7}$ is expressed as a decimal, find the 75^{th} digit to the right of the decimal point.

16. A horse drinks 2.14 milliliters of water per second. If the horse drinks at this same rate, find the number of milliliters of water the horse will drink in $\frac{1}{2}$ hour.

17. All ages in this problem are in years. Grandpa is now 4 times as old as Margie. Twelve years from now, Grandpa will be 3 times as old as Margie is then. Find the number of years in Grandpa's age now.

18. Find the smallest positive integer greater than 19269 that is an integral multiple of 330.

19. If eight times the sum of a number and eight is equal to 56, find the number.

20. Choosing from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, how many distinct sets of 5 different integers exist if exactly two members of each 5-member set are in the ratio 1:2?

2007 RA

Algebra I

Name ______________________

School ______________________

(Use full school name – no abbreviations)

______ Correct X **2** pts. ea. = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 4
2. 27.12 (Must be this decimal.)
3. 15
4. 8
5. 32000
6. $\frac{4}{3}$ (Must be this reduced improper fraction.)
7. $\frac{1}{2}$ (Must be this reduced common fraction.)
8. 21
9. 380
10. 29
11. 24
12. 15 (Hours optional.)
13. $\frac{12}{5}$ OR $2\frac{2}{5}$ OR 2.4
14. 77
15. 2
16. 3852 (Milliliters optional.)
17. 96 (Years optional.)
18. 19470
19. -1
20. 61

1. The degree measure of the supplement of $\angle ABC$ is $136°$. Find the degree measure of the complement of $\angle ABC$.

2. Find the degree measure of each angle of an equilateral triangle.

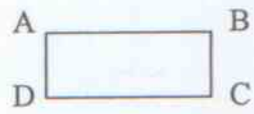
3. **(Always, Sometimes, or Never)** For your answer, write the ***whole word*** **Always, Sometimes,** or Never—whichever is correct.

   Two times the length of a diameter of a circle is more than the circumference of that circle.

4. In a parallelogram, the degree measures of three angles are represented by $3x+11$, $x+51$, and $7x-31$. Find the degree measure of one of the larger angles of the parallelogram.

5. **(Multiple Choice)** For your answer, write the **capital letter** which corresponds to the correct choice.

   In the diagram, if $ABCD$ is a parallelogram, then $\angle BCD$ **must** be congruent to:

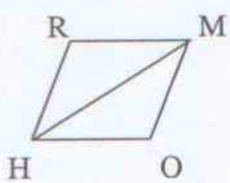


   A) $\angle BAD$
   B) $\angle ADC$
   C) $\angle ABC$

   Note: Be certain to write the correct capital letter for your answer.

6. Find the area of a rectangle with a base of length 30 and a diagonal of length 78.

7 In the diagram, RHOM is a rhombus, $\angle RHM = (3x-11)°$, and $\angle MHO = (x+27)°$. Find the degree measure of $\angle HRM$.



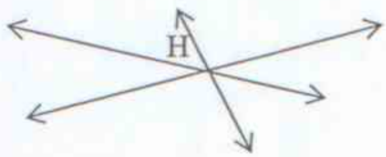
8 Given the points: $P(-4,1)$, $R(12,3)$, and $Q(5,10)$. How much farther is R from P than Q is from P? Express your answer as a **decimal** rounded to the nearest tenth.

9 A regular hexagon $ABCDEF$ is rotated $180°$ clockwise around the axis $\overleftrightarrow{AD}$. If $AD = 8$, then the volume of the solid generated by the rotation is $k\pi$. Find the value of k.

10 A segment of length 40 is divided in the ratio of $3:5$. Find the length of the shorter part.

11. The diagonals of a rhombus have lengths of 54 and 240. Find the absolute value of the numerical difference between the number of units in the perimeter of the rhombus and the number of square units in the area of the rhombus.

12. In the figure below, three lines intersect at H as shown. If two distinct rays, each of whose endpoints are at H, are selected at random, find the probability that the rays are opposite. Express your answer as a common fraction reduced to lowest terms.



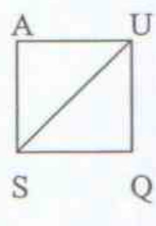
13. Two vertices of an isosceles right triangle are $A(0,0)$ and $B(4,0)$. If $\overline{AB}$ is a leg and all points of the other leg of the right triangle, except for one endpoint of that other leg, lie in Quadrant IV, find the remaining vertex. Express your answer as an ordered pair of the form (x, y).

14. In a circle with center at O, points B, A, C, and E lie on a circle in the order given and in a clockwise direction. $\overline{AE}$ is a diameter, minor arc $\widehat{BA} = 96°$, and minor arc $\widehat{AC} = 140°$. Point D lies on chord $\overline{BC}$ such that $\overline{AD} \perp \overline{BC}$. Find the degree measure of $\angle DAE$.

15. In right triangle ABC, the length of the hypotenuse is 9, and the sum of the lengths of the two legs is $3\sqrt{13}$. Find the area of triangle ABC.

16. Find the degree measure of one of the interior angles of a regular hexagon.

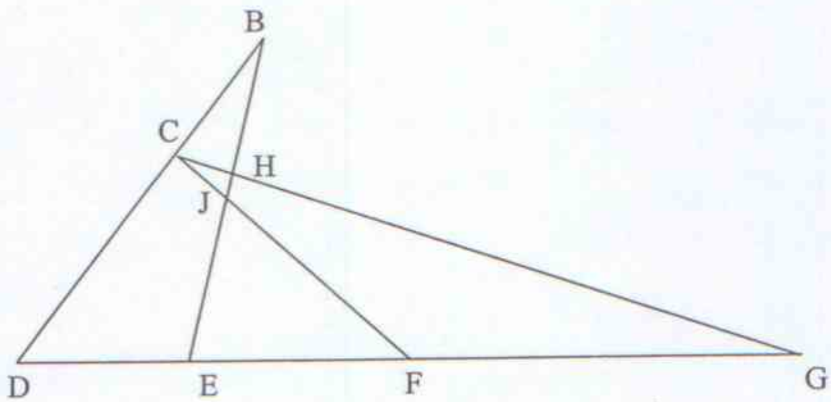
17 In square SQUA, $AU = 1.63x$, and $\angle ASU = (5.41x - 12)°$. Find the perimeter of square SQUA. Express your answer as a decimal rounded to the nearest tenth.



18. How many of the interior angles of a right triangle are acute angles?

19. Given ΔABC with $B(7,0)$ and $C(19,0)$. Point D lies on $\overline{BC}$ such that $\angle BAD \cong \angle CAD$. If $AB = 7$ and $AC = 8$, find the *x*-coordinate of point *D*. Express your answer as an improper fraction reduced to lowest terms.

20. In the diagram below, B, C, and D are collinear; D, E, F, and G are collinear; C, H, and G are collinear; B, H, J, and E are collinear; and C, J, and F are collinear. The length of the segment from B to E is 494, $BC : CD = 2:3$, and $DE : EF : FG = 2:3:4$. Find the length of the segment from H to J.



A

Geometry

Name R07

School ______

______ Correct X **2** pts. ea. = [ ]

School Code ______

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 46 (degrees optional)

2. 60 (degrees optional)

3. Never

4. 109 (degrees optional)

5. A

6. 2160

7. 88 (degrees optional)

8. 3.4 (must be this decimal)

9. 64

10. 15

11. 5988

12. $\frac{1}{5}$ (must be this reduced common fraction)

13. $(4, -4)$ (must be this ordered pair)

14. 22 (degrees optional)

15. 9

16. 120 (degrees optional)

17. 68.7 (must be this decimal)

18. 2

19. $\frac{63}{5}$ (must be this reduced improper fraction)

20. 32

1. Find the value of x such that $\sqrt{x-3} = 8$.

2. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

   If $3x + y^2 = 18$ and $x = 1$, then the values for y are:

   A) Both integral.
   B) 1 integral and 1 non-integral.
   C) Both imaginary.
   D) Both irrational.
   E) 1 rational and 1 irrational.

   Note: Be certain to write the correct capital letter as your answer.

3. Find the arithmetic mean (or average) of 18 and -14.

4. Solve the determinant equation for x: $\begin{vmatrix} 3 & 1 & 2 \\ -1 & x & 5 \\ 1 & 2 & 1 \end{vmatrix} = -24$.

5. Let $i = \sqrt{-1}$. The reciprocal of $\frac{2}{13} - \frac{3}{13}i$ can be written as $x + yi$ where x and y are real numbers. Find the value of $(x + y)$.

6. If $f = \{(2,3),(0,4),(4,5),(7,5)\}$ and if $g = \{(5,7),(2,4),(4,2)\}$, find the value of $f(g(4)) + g(f(7))$.

7. Let $i = \sqrt{-1}$. If $z = a + bi$ where a and b are real numbers, let $\bar{z}$ be defined as the complex conjugate of z. If $z + 3\bar{z} = 84 - 16i$, find the value of z. Express your answer for z in $a + bi$ form.

8. If x varies inversely as y, and if $x = 8$ when $y = 5$, find the value of x when $y = 13$. Express your answer as an improper fraction reduced to lowest terms.

9. Find the value of k for which $\log\left(\frac{x+y}{k}\right) = \frac{\log(x)}{3} + \frac{\log(y)}{3} + \frac{\log(x+y)}{3}$ for all positive values of x and y for which $x^3 + y^3 = 340xy(x+y)$.

10. In the number base problem, $12_{(x+y)} = 9_{ten}$. Find the value of $24_{(x+y)}$. Express your answer in base ten.

11. Given 4 fractions: $\frac{2x^2+3x-5}{3x^2-x-2}, \frac{x^2-x-6}{2x^2-x-21}, \frac{x^2-16}{(x+4)(x-4)}, \frac{x^2+x-2}{(x+1)(x-1)}$
Assuming non-zero denominators, if one of these fractions is selected at random, find the probability that the fraction will reduce over the integers. Express your answer as a common fraction reduced to lowest terms.

12. If $x = \sqrt{23+\sqrt{23+\sqrt{23+\sqrt{23+\cdots}}}}$, then $x = \frac{k+\sqrt{w}}{2}$ where k and w are positive integers. Find the value of $(k+w)$.

13. If $2x+1$, $7x-119$, and $5x-57$ form an arithmetic progression in that order, find the value of x.

14. Find the sum of the two distinct values of k for which the cubic polynomial equation $x^3-9x^2+15x+k=0$ has a double root (meaning two of the three roots are equal) when solved for x.

15. Find the value of k if $2x-3$ is a factor of $8x^3-16x^2+kx+3k$.

16. Find the ordered pair of the form (x, y) that represents the point that is the reflection of the point $(5,-7)$ with respect to the line $y = x$. Express your answer as an ordered pair of the form (x, y).

17. Let x, y, and z be three integers such that $x < y < z$ and such that $y-x = z-y$. If the product of these three integers is a prime positive integer, find the value of x.

18. The line $y = 3x+p$ is tangent to the parabola $y = 2x^2-4x+2$. Find the value of p. Express your answer as an **exact decimal**.

19. Find the fifty-sixth term of the arithmetic sequence: $1, 3, 5, 7, 9, \cdots$,

20. If x and y are each positive integers and each is less than 100, how many distinct ordered pairs (x, y) exist such that (x, y) lies on one of the bisectors of the angles formed by the lines $5x-12y+80=0$ and $12x+5y-2=0$?

A

Algebra II

Name R07

School

_____ Correct X **2** pts. ea. = [ ]

School Code

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 67

2. D

3. 2

4. 4

5. 5

6. 10

7. $21+8i$ or $8i+21$

8. $\frac{40}{13}$ (must be this reduced improper fraction)

9. 7

10. 18 or 18_{ten} or 18_{10}

11. $\frac{3}{4}$ (must be this reduced common fraction)

12. 94

13. 26

14. 18

15. 2

16. $(-7, 5)$ (must be this ordered pair)

17. -3

18. -4.125 (must be this decimal)

19. 111

20. 6

1. Find the real value of x such that $\sqrt[3]{x-2} = 6$.

2. If (r,s), $(5,8)$, and $(10,-4)$ represent vectors, find the ordered pair (r,s) such that $(r,s)+(5,8)=(10,-4)$.

3. If A is a radian measure such that $\tan(A)=0.9143$ and such that $\pi < A < \frac{3\pi}{2}$, find the value of A. Express your answer as a decimal rounded to 4 significant digits.

4. The absolute value of the number of radians in the angle of the rotation of the minute hand of a clock during a time period of 72 minutes can be expressed as $k\pi$. Find the value of k. Express your answer as an exact decimal.

5. For all real-valued x, $2(\sin(2x)\cos(2x)) = A(\sin(Bx))$. If A and B are positive integers, find the value of $(A+B)$.

6. For how many distinct real values of x does $\frac{x}{2}+\frac{x}{4}=\frac{2x}{6}$?

7. The vertex of an angle is $(2,0)$. A point on the initial ray is $(6,0)$, and a point on the terminal ray is $(5,7)$. Find the cotangent of the angle. Express your answer as a common fraction reduced to lowest terms.

8. Find the units' digit of 7^{1000}.

9. In Triangle ABC, $AB=6$, $BC=12$, and $\angle ABC = 90°$. D lies on $\overline{AC}$ such that $\angle ABD \cong \angle CBD$. Find BD.

10. If $f(x)=19x-6$, $g(x)=ax+18$, and $f(g(7))=2065$, find the value of a.

11. From three girls and two boys, two persons are selected at random to form a two person math team. Find the probability that the two persons selected were a girl and a boy. Express your answer as a common fraction reduced to lowest terms.

12. An ellipse has its vertices at the points $(6,-3)$ and $(-4,-3)$ and has the endpoints of its minor axis at the points $(1,-5)$ and $(1,-1)$. The eccentricity of this ellipse can be expressed as $\frac{\sqrt{k}}{w}$ where k and w are positive integers. Find the minimum value of $(k+w)$.

13. A set of experimental measurements of the freezing point of an unknown liquid yield an arithmetic mean of 26.24° Celsius with a standard deviation of 1.48° Celsius. If this same experiment had been conducted on the Fahrenheit scale, find the sum of the arithmetic mean and the standard deviation in Fahrenheit degrees. Express your answer as an **integer** rounded to the nearest Fahrenheit degree.

14. Let a sequence $\{u_n\}$ be defined by $u_1 = 5$ and the relation $u_{(n+1)} - u_n = 5 + 7n^2$, $n = 1, 2, 3, 4, \ldots$. If this sequence is expressed as a cubic polynomial such that the coefficient of n^3 is $\frac{7}{3}$, find the coefficient of n. Express your answer as an improper fraction reduced to lowest terms.

15. It is known that x varies directly as y and inversely as the square of z. If $x = \frac{2}{9}$ when $y = 10$ and $z = 3$, find the value of x when $y = 20$ and $z = 2$.

16. The area of a circle with center at $(2,5)$ and passing through $(-3,17)$ is $k\pi$. Find the value of k.

17. Let $x \in \{-9,-8,-7,-6,-5,-4,-3,-2,-1,1,2,3,4\}$. If f is a real-valued function such that $f(x) = \sqrt{\frac{x+2}{x-1}}$, find the sum of all possible distinct values of x that are in the domain of f.

18. The transformations necessary to produce the graph of $y = 2x^2 + 24x - 1$ from the graph of $y = x^2$ are a horizontal shift 6 to the left, a vertical stretch by a factor of 2, and a vertical shift k downward. Find the value of k.

19. If A is a radian measure such that $\sin^4(A) = \frac{1}{81}$, how many distinct solutions are there such that $0 \le A \le \pi$?

20. Let $n(A)$ refer to the cardinal number of A. If $n(A) + n(B) + n(C) + n(D) = 115$, $n(A \cap B) + n(A \cap C) + n(A \cap D) + n(B \cap C) + n(B \cap D) + n(C \cap D) = 99$, $n(A \cap B \cap C) + n(A \cap B \cap D) + n(A \cap C \cap D) + n(B \cap C \cap D) = 41$, and $n(A \cap B \cap C \cap D) = 7$, find $n(A \cup B \cup C \cup D)$.

A
Pre-Calculus

Name R07

School ____________

______ Correct X 2 pts. ea. = [ ]

School Code ____________

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 218

2. $(5,-12)$ (must be this ordered pair)

3. 3.882 (must be this decimal)

4. 2.4 (must be this decimal)

5. 5

6. 1

7. $\frac{3}{7}$ (must be this reduced common fraction)

8. 1

9. $4\sqrt{2}$

10. 13

11. $\frac{3}{5}$ (must be this reduced common fraction)

12. 26

13. 82 (degrees and/or Fahrenheit optional)

14. $\frac{37}{6}$ (must be this reduced improper fraction)

15. 1

16. 169

17. –35

18. 73

19. 2

20. 50

FROSH-SOPH EIGHT PERSON TEAM COMPETITION
ICTM REGIONAL 2007 DIVISION A

NO CALCULATORS

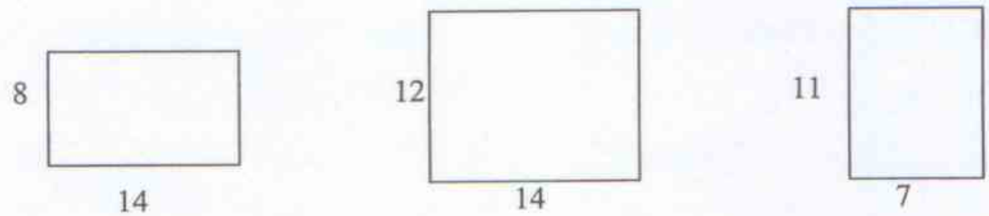
1. By definition, what one word description is given to a triangle that has sides of three different lengths?

2. **(Always, Sometimes, or Never)** For your answer, write the whole word **Always, Sometimes,** or **Never**—whichever is correct.

   The two distinct diagonals of a convex quadrilateral intersect each other.

3. Find the perimeter of a rhombus if the lengths of the diagonals of the rhombus are respectively 6 and 8.

4. The dimensions (in feet) of three rectangular floors are shown below. If one of the floors is selected at random and it costs 13 cents per square foot to paint the floor, find the probability that the cost of painting the floor selected is more than \$15.00. Express your answer as a common fraction reduced to lowest terms.



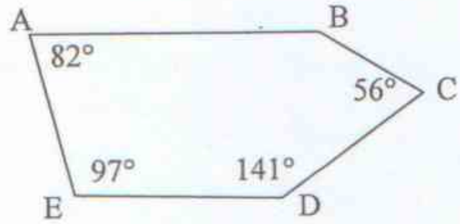
5. If $0.2x+0.5y=0.6$ and $0.4x-0.3y=1.72$, find the value of x.

6. Assume that A and B are two distinct points on a perfectly straight highway and that A is k miles from B. A man leaves point A at 12:42 P. M. and heads toward B at a constant rate of 40 mph. At 12:57 P. M. that same day, the man's wife leaves point B and heads toward A at a constant rate of 50 mph. If the two meet exactly halfway from A to B, find the value of k.

7. If $\left(x^3+13\right)^2\left(x-8\right)^2$ is expressed as a polynomial in x, find the degree of that polynomial.

8. Find the sum of all integers greater than twenty and less than one thousand whose number base five representation consists of all identical digits. Express your answer for the sum as a **base ten numeral**.

9. In the figure to the right with degree measures as shown, find the degree measure of $\angle$ ABC.



10. If $x^2-13x+2=0$, find the sum of the reciprocals of the roots.

11. Points A, B, C, and D lie in plane f. No three of the four points are collinear. From point P outside the plane, $\overline{PA}$ is drawn such that $\angle PAB=(5x+y+2)°$, $\angle PAC=(2x+3y-5)°$, and $\angle PAD=(8x+w-28)°$. For what value of w is $\overline{PA}\perp f$?

12. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

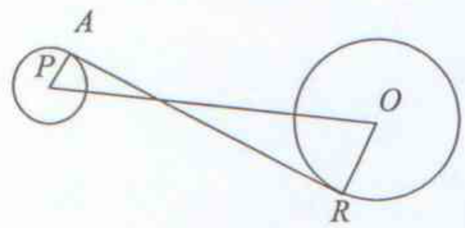
How much water should be evaporated from x gallons of a solution that is 30% salt to obtain a solution that is 50% salt?

A) $\frac{2}{5}x$
B) $\frac{2}{3}x$
C) $\frac{1}{3}x$
D) $\frac{3}{10}x$
E) $\frac{1}{2}x$

Note: Be certain to write the correct capital letter as your answer.

13. One light has a 7 watt bulb; a second light has a 40 watt bulb; a third light has a 75 watt bulb. Two of these lights are selected at random and turned on. The other light is left off. If the two lights selected are left on for 12 hours, find the probability that more than 1 kilowatt-hour of electric energy will be used for these 12 hours. Express your answer as a common fraction reduced to lowest terms.

14. In the diagram, P and O are centers of the circles. $\overline{AR}$ is a common internal tangent segment with points of tangency at A and R. $PA = 4$, $OR = 7$, and $PO = 13$. Find AR.



15. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

Which pair of numbers would represent the degree measures of two angles that are **neither** complementary nor supplementary?

A) 45 and 45 B) 90 and 90 C) 50 and 50
D) 20 and 70 E) 40 and 140

Note: Be certain to write the correct capital letter as your answer.

16. All ages in this problem are in years. A man is three times as old as his wife was when the man was as old as his wife is now. The man is now 48. Find the number of years in the present age of his wife.

17. **(Always, Sometimes, or Never)** For your answer, write the whole word **Always, Sometimes,** or **Never**—whichever is correct.

If k is a positive integer such that the slopes of two consecutive sides of a parallelogram are respectively k and $-\frac{1}{k}$, then the parallelogram is a rectangle.

18. **(Always, Sometimes, or Never)** For your answer, write the whole word **Always, Sometimes,** or **Never**—whichever is correct.

If a theorem is true, then the converse of that theorem is true.

19. The lengths of the sides of a right triangle are all integers. The first side has a length that is an integral multiple of 13, and one of the other sides has a length that is prime and is 50 more than the length of the first side. Find the smallest possible perimeter of such a triangle.

20. The sum of the cubes of the roots of a quadratic equation is equal to 36 times the product of those roots. If the sum of the roots of that same quadratic equation is 12, the larger of the two roots of the quadratic equation is equal to $k + w\sqrt{f}$ when expressed in simplest radical form. Find the value of $(k + w + f)$.

A

Fr/So 8 Person

School ____R07____

______ Correct X **5** pts. ea. = [ ]

School Code ____________

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. Scalene

2. Always

3. 20

4. $\frac{1}{3}$ (must be this reduced common fraction)

5. 4

6. 100 (miles optional)

7. 8

8. 2675 or 2675_{ten} or 2675_{10}

9. 164 (degrees optional)

10. $\frac{13}{2}$ or $6\frac{1}{2}$ or 6.5

11. 14

12. A

13. $\frac{1}{3}$ (must be this reduced common fraction)

14. $4\sqrt{3}$

15. C

16. 32 (years optional)

17. Always

18. Sometimes

19. 208

20. 11

NO CALCULATORS

1. Find the value of x such that $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 8 & 9 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ x & 13 \end{bmatrix}$.

2. Is the vector $(-3,8)$ perpendicular to the vector $(12,4.5)$? Answer **Yes** or **No,** whichever is correct.

3. **(Always, Sometimes, or Never)** For your answer, write the <u>whole word</u> **Always, Sometimes,** or **Never**—whichever is correct.

   If $[C]$ and $[D]$ are two matrices, then $[C][D]=[D][C]$.

4. A line passes through $(3,6)$, $(-4,4)$, $(-11,k)$, and $(w,8)$. Find the value of $(k+w)$.

5. If an expression is selected at random from the following four expressions: $4x-8,\ x^2+5x-6,\ x^2+3x+4,\ x^2-11x+30$, what is the probability that the expression selected will factor over the integers? Express your answer as a common fraction reduced to lowest terms.

6. In what quadrant is cosine positive and tangent negative? For your answer, write a **Roman numeral**.

7. Find the cosine of the second largest angle of a triangle that has sides with lengths of 10, 14, and 16. Express your answer as a common fraction reduced to lowest terms.

8. If x is a real number, find the sum of the squares of all distinct x such that $x^{240}-4096^{80}=0$.

9. If one of the members of the set $\{58,304,430,510,-42\}$ is selected at random and substituted for x in the trig function $\sin(x°)$, find the probability that $\sin(x°)>0.5$. Express your answer as a common fraction reduced to lowest terms.

10. If $f(x)=x^2-2$ and $g(x)=2x+1$, find the value of $f(g(5))$.

11. Let $P=2^{(\frac{1}{2})}\bullet 2^{(\frac{2}{4})}\bullet 2^{(\frac{3}{8})}\bullet 2^{(\frac{4}{16})}\bullet 2^{(\frac{5}{32})}\bullet\bullet\bullet 2^{(\frac{n}{2^n})}$. If $n=25$, then $P=2^{(\frac{2^w-f}{2^{25}})}$. If w and f are positive integers, find the smallest possible value of $(w+f)$.

NO CALCULATORS

12. If x is a positive integer, find the sum of all distinct values of x such that $\sqrt{x-6}$ is an imaginary number.

13. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

In how many non-congruent scalene triangles does the cosine of one angle equal the sine of a second angle?

A) 0 B) 1 C) 2 D) In every right triangle but in no obtuse triangle
E) None of the choices (A—D) is correct.

Note: Be certain to write the correct capital letter as your answer.

14. One of the numbers of the set $\{120,130,140,150\}$ is selected at random. Find the probability that the number selected is the degree measure of an obtuse angle formed at the intersection of the two distinct altitudes drawn to the legs of an isosceles triangle with a vertex angle of $80°$.

15. Find the sum of the odd integers that are greater than zero and less than 68.

16. In Triangle ABC, $AB = 80$, $BC = 99$, and $\angle ABC = 60°$. Find the length of $\overline{AC}$.

17. In a room with exactly 6 distinct persons, exactly 12 handshakes between 2 distinct persons are made. No two of these handshakes took place between the same 2 persons. Of the persons in the room, exactly two persons shook hands with exactly three other persons. Two distinct persons from these 6 persons are selected at random. Find the probability that those two distinct persons had made one of the 12 handshakes. Express your answer as a common fraction reduced to lowest terms.

18. Find the value of $\left(\log_c(a)\right)\left(\log_d(b)\right)\left(\log_a(d)\right)\left(\log_b(c)\right)$.

19. A drawer contains an odd number of orange socks and an even number of blue socks. Two socks are drawn at random (without replacement). Find the least number of orange socks and the least number of blue socks that could be in the drawer such that the probability that both socks drawn were orange would be $\frac{1}{2}$.

Express your answer as an ordered pair of the form $(\#orange, \#blue)$.

20. Three positive integers are in arithmetic progression. Let S be the sum of these 3 positive integers, and let P be the product of these 3 positive integers. If $P < 1000$ and if P has exactly 8 positive integral divisors, find the sum of all possible distinct values of S.

A

Jr/Sr 8 Person

School R07

______ Correct X **5** pts. ea. = [   ]

School Code __________

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 11
2. Yes
3. Sometimes
4. 12
5. $\frac{3}{4}$ (must be this reduced common fraction)
6. IV (must be this Roman numeral)
7. $\frac{1}{2}$ (must be this reduced common fraction)
8. 512
9. $\frac{2}{5}$ (must be this reduced common fraction)
10. 119
11. 53
12. 15
13. E
14. 0
15. 1156
16. 91
17. $\frac{4}{5}$ (must be this reduced common fraction)
18. 1
19. $(15,6)$ (must be this ordered pair)
20. 117

1. Find the value of $2.016^{(-5.048)}$.

2. Squares A, B, and C, and D have sides of respective lengths 6, 6.8, 12, and 8.6. If one of the four squares is selected at random, find the probability that the area of that square is more than 47.1. Express your answer as a **common fraction reduced to lowest terms.**

3. Find the value of $\cos(28°17')$.

4. The expression $y = 1.864x^3 - 56.24x^2 + 68.87x - 102.1$ has a local minimum between $x = 14.98$ and $x = 22.11$. Find the y-coordinate of this local minimum.

5. Solve for x: $5.247x - (2.789 - 1.603x) = 18.89$.

6. If x and y are real, and $x^2 + y^2 = 3$, find the maximum value of $(x+y)^3$.

7. $T = 2\pi\sqrt{\frac{L}{32}}$ is a formula for the period T (in seconds) of a pendulum whose length is L feet. If the period for a pendulum is 1.389 seconds, find the number of feet in the length of the pendulum.

8. Let the two lines whose equations are respectively $2x + 6y = 15$ and $y = 4x + 5$ intersect at point A. A circle with center at point A intersects one of the given lines at a point B and intersects the other of the given lines at a point C. If the radius of this circle is 12.42, find the largest possible value of the area of the sector bounded by minor arc $\overset{\frown}{BC}$ and radii $\overline{AB}$ and $\overline{AC}$.

9. The area of a square is 568.6. Find the area of the circle that is circumscribed about the square.

10. Find the perimeter of an equilateral triangle whose altitude has a length of 18.21.

11. Find the degree measure of the acute angle of elevation of the sun when a 40-foot mast casts a 30-foot shadow.

12. In ΔABC, $AB = 98.45$, $BC = 54.62$, and $\angle ABC = 51.16°$. Find the area of ΔABC.

13. Given: $A(-1,10)$ and $B(3,2)$. Point $C(x,y)$ lies on the perpendicular bisector of $\overline{AB}$ and is a distance of 62 from the midpoint of $\overline{AB}$. If $x > 0$ and $y > 0$, find the value of $(x+y)$.

14. From the chart shown, find the fraction of employees that earned less than \$310.00 per week but at least \$260.00 per week. Express your answer as a **common fraction reduced to lowest terms**.

WAGES	NUMBER OF EMPLOYEES
\$250.00-\$259.99	12
260.00-269.99	11
270.00-279.99	26
280.00-289.99	19
290.00-299.99	12
300.00-309.99	13
310.00-319.99	6

15. In Triangle ABC, $AB = 245$, and $\angle ABC = 60°$. If all sides of Triangle ABC have integral lengths, find the smallest possible perimeter of Triangle ABC such that the perimeter is greater than 2200. Express your answer for the perimeter as an **exact integer**.

16. If $x = 1.673$, find the value of $2x^{(-\frac{1}{3})}$.

17. In a circle with an area of 753.8, the length of minor arc $\overset{\frown}{CD}$ exceeds the length of chord $\overline{CD}$ by 0.06311. Find, to the nearest hundredth of a degree, the degree measure of minor arc $\overset{\frown}{CD}$. Express your answer as a **decimal**.

18. Find the value of $\sqrt[3]{\dfrac{4.246^{\pi}}{\sqrt{5.063}}}$.

19. If $(3x)^{(-\frac{4}{5})} = 1.887$, find the value of x if $x > 0$.

20. Find the absolute value of the shortest distance from $(2,5)$ to any point on the graph of the ellipse whose center is at $(0,0)$, whose major axis is vertical with a length of 8 and whose minor axis is horizontal with a length of 6.

A

Calculator Team

School ____R07____

____ Correct X 5 pts. ea. = [ ]

School Code ____________

Note: **All answers must be written legibly. Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.**

1. 0.02904 or 2.904×10^{-2}
2. $\frac{1}{2}$ (must be this reduced common fraction)
3. 0.8806 or 8.806×10^{-1}
4. -6323 or -6.323×10^{3}
5. 3.165 or 3.165×10^{0}
6. 14.70 or 1.470×10 or 1.470×10^{1} (trailing zero is necessary)
7. 1.564 or 1.564×10^{0}
8. 127.1 or 1.271×10^{2}
9. 893.2 or 8.932×10^{2}
10. 63.08 or 6.308×10 or 6.308×10^{1}
11. 53.13 or 5.313×10 or 5.313×10^{1} (degrees optional)
12. 2094 or 2.094×10^{3}
13. 90.18 or 9.018×10 or 9.018×10^{1}
14. $\frac{9}{11}$ (must be this reduced common fraction)
15. 2205 (must be this integer)
16. 1.685 or 1.685×10^{0}
17. 26.42 (must be this decimal) (degrees optional)
18. 3.469 or 3.469×10^{0}
19. 0.1507 or 1.507×10^{-1}
20. 1.555 or 1.555×10^{0}

1. A radius of an old circle is increased by 2 units and becomes the radius of a new circle. The area of this new circle is 25.34π. Find the circumference of the old circle. Express your answer as a **decimal** rounded to the nearest hundredth.

2. The sum of 6 consecutive positive integers is 87. Find the smallest of the 6 integers.

3. How many of the following nine numbers are integral multiples of 11?
$$\{88, 121, 275, 913, 583, 264, 301, 16258, 425\}$$

4. Let k be a positive integer such that k, $k+8$, and $k+16$ are the lengths of the three sides of a right triangle. Find the value of k.

5. The lengths of the diagonals of 3 squares are respectively 8, 10, and 14. The ratio of the area of the smallest square to the area of the largest square can be expressed as $k:w$ where k and w are positive integers. Find the smallest possible value of $(k+w)$.

6. Given the points: $F(-2,1)$, $G(10,4)$, $H(x,y)$, and $J(1,8)$. If $FGHJ$ is a parallelogram, find the value of $(x+y)$.

7. Find the sum of all distinct values of three digit integers greater than 500 if each three digit integer is a square of an integer.

8. If $k\sqrt{32} + \sqrt{200} - \sqrt{1682} = \sqrt{2}$ and if $w\sqrt{243} + \sqrt{192} - \sqrt{1875} = \sqrt{3}$, find the value of $(k+w)$.

9. In ΔABC with $A(-2,-1)$, $B(-2,5)$, and $C(6,-1)$, find the coordinates of the point that is equidistant from A, B, and C. Express your answer as an **ordered pair** of the form (x,y).

10. Let x represent the number of real solutions of the equation $2y^3+15y^2-176y+84=0$. Find the value of determinant: $\begin{vmatrix} 1 & -4 & 5 \\ 6 & x & 2 \\ 7 & 5 & 1 \end{vmatrix}$