1. The numerator of a fraction is three less than the denominator. If the numerator is increased by 4 and the denominator is decreased by one, the resulting fraction is \( \frac{3}{2} \). Find the original common fraction. Write your answer as that common fraction.

2. Solve for \( x \):
\[
\frac{1}{x} + \frac{2}{x} + \frac{5}{x} = 32
\]

3. The vertex of \( y = x^2 + bx + c \) is at \((2, -3)\). Find the value of \( b + c \).

4. The fraction \( \frac{9 - 4x^2}{x^2 - x - 20} \) is a real number for all \( x \) except for \( k \) and \( w \). The given fraction is zero when \( x = p \) and when \( x = m \). Find the value of the product \( kwpm \).

5. Find the real solution for \( x \):
\[
2^{3x+1} = 4^{x+3}
\]

6. If the graph of \( y = ax + b \) is perpendicular to the graph of \( 4x + 2y = 10 \) and contains the point \((-4, 6)\), find the product \( ab \).

7. How many liters of a 10\% solution of acid must be mixed with a 50\% solution of acid to make 50 liters of a 40\% solution of acid? Express your answer as an exact decimal.

8. If \( f(x) = 2x + 1 \) and \( m(x) = 3x - 2 \), then \( \frac{f(2x - 1) - m(x + 1)}{f(x) + 2m(x)} \) can be written, in simplest form, as \( \frac{x + k}{wx + p} \). Find the value of \( k + w + p \).

9. If \( x \) is a randomly chosen solution to \( |4x - 8| < 20 \), find the probability that \( x \) is also a solution of \( 4 - 3x < -8 \). Express your answer as a common fraction reduced to lowest terms.

10. Given \( a > 0, b > 0, c > 0 \), find \( x + y + z \) if
\[
\frac{\sqrt[6]{a^3 b^2 c^8} - 6}{\sqrt[3]{a^4 b^2 c^8}} = a^x b^y c^z.
\]
11. If \( a \oplus b = a^2 + 3ab \), compute \( (\sqrt{3} \oplus 2\sqrt{3}) \oplus 4 \).

12. The average (mean) age of a group of ten 8th graders is 14, while a group of 12th graders has an average age of 18. If the average age of the entire group is 17, how many 12th graders are in the group?

13. A car travels at 30 mph. for an unknown number of miles and then immediately travels for 2 hours at 40 mph. If the average speed for the entire trip was 34 mph, how many miles did the car travel at 30 mph?

14. The longer leg of a right triangle exceeds the shorter leg by 14, and the hypotenuse is 6 more than twice the shorter leg. Find the triangle’s perimeter.

15. When \( 3ab^2 - 2bc^2 = \frac{a}{c} + 2abc^2 + abc \) is solved for \( a \), the result can be written in the form \( \frac{kbc^3}{mb^2c + nbc^2(2c + 1) - 1} \).

Assuming non-zero denominators, find the value of \( k + m + n \).

16. A sale item at Wards was sold for $69.30 including 5% sales tax. The original price of the item had been reduced by 20% to a new price and then that new price was reduced by 25% just before it was sold. What was the original price, not including sales tax?

17. If \( y < \) two hundred, find the sum of all distinct \( y \) such that \( x \) and \( y \) are positive integers representing number bases and, when expressed in proper form, \( 215_x = 18_y \). Express your answer as a base ten numeral.

18. A number \( x \) varies inversely as the square of \( y \) and directly as \( z \). If \( y \) is doubled and \( z \) is tripled, then \( x \) will be multiplied by \( w \). Find the value of \( w \).

19. Factor over the integers into two trinomials: \( x^2 + 3xy - 5x + 2y^2 - 7y + 6 \)

20. It takes Ann 4 fewer minutes to clean the chalkboards than it takes Bill. They started cleaning the chalkboards together at 9:32 A.M. They should have finished the job in \( 18 \frac{18}{19} \) minutes, but after \( 18 \frac{18}{19} \) minutes, the job was only \( \frac{5}{6} \) done because Ann worked continuously, while Bill quit early. Find the number of minutes after 9:32 A.M. that Bill quit working. Express your answer as an improper fraction reduced to lowest terms.
Algebra I

Correct × 2 pts. ea. =

Note: All answers must be written legibly stated in the Contest Manual. Exact specified in the question. No units of

1. \( \frac{2}{5} \) (must be this exact fraction)

2. \( \frac{1}{4} \) or 0.25 or .25

3. -3

4. 45

5. 5

6. 4

7. 12.5 (must be this decimal) (liters optional)

8. 3

9. \( \frac{3}{10} \) (must be this reduced common fraction)

10. -3

11. 693

12. 30

13. 90 (miles optional)

14. 60

15. 4

16. 110 (dollars optional)

17. 478 (base ten optional)

18. \( \frac{3}{4} \) or 0.75 or .75

19. \( (x + y - 2)(x + 2y - 3) \) or equivalent simplified trinomial factors in any order

20. \( \frac{700}{57} \) (must be this reduced improper fraction) (minutes optional)